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STRUCTURAL APPROACH IN PROBLEMS OF THE LIMIT EQUILIBRIUM OF BRITTLE SOLIDS WITH STRESS CONCENTRATORS

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Versions of the formulation of a two-dimensional fracture criterion are discussed. Possible methods for determining the value of the structural fracture parameter are analyzed. Theoretical estimates are compared with experimental data and results obtained using alternative criteria.

Key words: *crack, hole, cavity, structural criterion, critical load.*

Introduction. Recently, the so-called nonlocal fracture criteria (see [1] and the bibliography therein), in particular, the integral criterion, or the mean-stress criterion have been expensively used to estimate the strength of materials with stress concentrators. The idea of averaging stresses on a certain interval of length d ahead of a crack tip was proposed by Neuber [2] and Novozhilov [3]. The discrete Neuber–Novozhilov criterion in a one-dimensional formulation has been extensively used by various researchers. A two-dimensional mean-stress criterion for a disk-shaped opening mode crack was proposed in [4].

The presence of the averaging parameter d in the criterion implies that the fracture process has its own structure, which generally may not be related to the structure of the material. By virtue of this, following [5], we will call the mean-stress criterion the structural criterion and the Neuber–Novozhilov approach the structural approach.

Advantages of the structural criterion are simplicity, applicability to singular defects (cracks and angular notches) and regular defects (holes and cavities), and the possibility of using approximate and exact analytical solutions of elastic problems. In the latter case, as the defect size gradually decreases, passage to the limit of a defect-free material occurs (see also [6]). In the formulations of the criterion considered in the present paper, we use only two material constants: the tensile strength limit σ_c and the static fracture toughness $K_{\rm Ic}$, both mechanical characteristics being determined from the results of standard tests.

Unlike in the one-dimensional formulation of the structural criterion, in the two-dimensional case there are some difficulties due to the necessity of choosing a particular value of the structural parameter d for a quantitative estimation of the dimensional critical loads. For media with smooth defects (notches, holes, and cavities), determination of the parameter d remains an important problem in the one-dimensional criterion, too. In the present paper, computational schemes of solving these problems are analyzed.

1. One-Dimensional Fracture Criterion. We consider an elastic homogeneous isotropic plane weakened by a rectilinear central crack of length $2l$. The Ox axis, whose origin is at the center of the crack, passes through the line of location of the crack. At infinity, the plane is extended by a uniform load p in the Oy direction perpendicular to the crack.

For the fracture stress on the continuation of the crack, the approximate solution has the form

$$\sigma_y = K_{\rm I}/\sqrt{2\pi(x-l)} + O(1), \qquad x-l\to 0, \quad x>l, \tag{1.1}$$

where $K_{\rm I} = p\sqrt{\pi l}$ is the stress intensity factor.

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The exact solution is given by

$$\sigma_y = px/\sqrt{x^2 - l^2}\,, \qquad y = 0, \quad x > l. \tag{1.2}$$

To the asymptotic solution (1.1) we apply the structural criteria

$$\frac{1}{d}\int\limits_{l}^{l+d} \sigma_y(x)\,dx = \sigma_c, \tag{1.3}$$

where d is the structural fracture parameter and σ_c is the tensile strength limit of the material. Assuming that fracture occurs if the equality $K_{\rm I} = K_{\rm Ic}$ is satisfied (the Irwin criterion), we obtain the quantity

$$d = 2K_{\rm Ic}^2/(\pi\sigma_c^2) \tag{1.4}$$

and the critical load

$$p_* = K_{\rm Ic}/\sqrt{\pi l}\,. \tag{1.5}$$

Substitution of the exact solution (1.2) into criterion (1.3) yields an expression for the critical load that is valid for a crack of any length:

$$p_* = \sigma_c/\sqrt{1 + 2l/d}\,. \tag{1.6}$$

It is of interest to compare the quantity p_* with the critical load calculated by the Griffith and Leonov–Panasyuk criteria and with experimental results. The dependence of the fracture load on the length of a central crack in a glass plate loaded uniaxially at the edges by a uniform load along the normal to the fracture plane was studied experimentally in [7]. The mechanical characteristics of silicate glass are as follows: tensile strength limit $\sigma_c = 39.2$ MPa [8], Poisson's ratio $\nu = 0.24$, elastic modulus $E = 67$ GPa, and specific surface fracture energy $\gamma = 2.1\cdot 10^{-6}$ J/mm^2 ($2.1\cdot 10^{-3}$ MPa $\cdot$ mm); the fracture toughness is determined using the material characteristics ν, E, and γ [7] and is equal to $K_{\rm Ic} = 0.546$ MPa $\cdot$ m $^{1/2}$.

The formulas for the critical load have the following form [9]:

— for the Griffith criterion,

$$p_* = \sqrt{2E\gamma/(\pi(1-\nu^2)l)}\,; \tag{1.7}$$

— for the Leonov–Panasyuk criterion,

$$p_* = (2/\pi)\sigma_c \arccos\,[\exp\,(-\delta_c/(8\sigma_c c l))]. \tag{1.8}$$

In (1.8), $c = (1-\nu^2)/(\pi E)$ and $\delta_c = 2\gamma/\sigma_c$ is the critical crack opening, which depends on the specific fracture energy and the strength of the material.

Results of calculation using formulas (1.5)–(1.8) and the experimental data of [7] are presented in Fig. 1. It is evident that in the examined range of crack lengths, the criteria give identical estimates of the critical load. Differences are observed only for $2l < 1$ mm.

We note that for $l \to 0$, the Griffith and Irwin criteria give an infinitely large value of the critical load, whereas the Leonov–Panasyuk criterion and the structural criterion imply that $p_* \to \sigma_c$. In other words, a plate with a crack of zero length has the strength of a defect-free material. However, the rate of approach of the critical load to the strength limit is different:

— for criterion (1.6),

$$p_*(l) = \sigma_c(1 - l/d) + O(l^2);$$

— for criterion (1.8),

$$p_*(l) = \sigma_c + O(0).$$

Therefore, for the indicated material, the critical load determined according to (1.8) for crack lengths of 0–0.03 mm is almost unchanged and is equal to σ_c. The structural parameter for glass is $d = 0.124$ mm.

Equating the critical loads determined by the Irwin (1.5) and Griffith (1.7) criteria, we obtain the well-known relationship between the specific surface fracture energy γ and the fracture toughness $K_{\rm Ic}$ (for planar deformation):

$$\gamma = (1-\nu^2)K_{\rm Ic}^2/(2E). \tag{1.9}$$

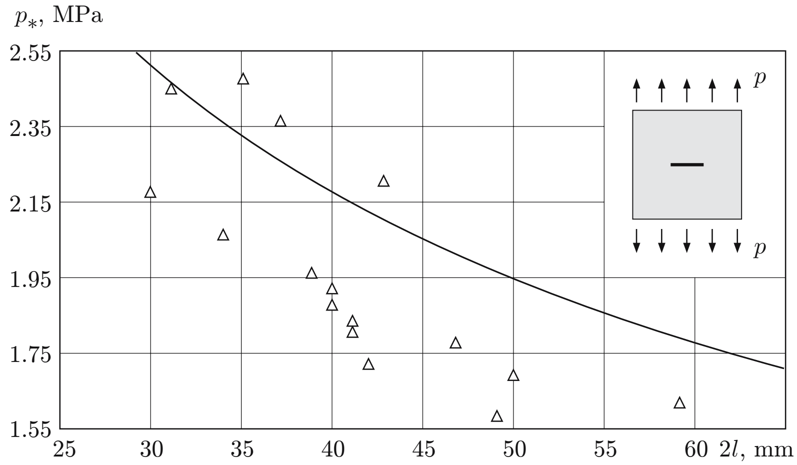

Fig. 1. Critical load versus crack length: the curve refers to the calculation results and the points refer to the experimental data [7].

We now determine the dependence of the structural parameter d on the specific fracture energy γ. From (1.4), we have

$$K_{\rm Ic}^2 = \pi d \sigma_c^2/2. \tag{1.10}$$

Substitution of (1.10) into (1.9) yields

$$d = 4E\gamma/(\pi(1-\nu^2)\sigma_c^2),$$

which almost coincides with the estimate [10]

$$d = 4E\gamma/(3(1-\nu^2)\sigma_c^2).$$

Using polar coordinates (r, θ), we consider the uniaxial extension of an infinite elastic plate with a circular hole of radius a. If a uniform tensile stress p applied at infinity acts in the direction $\theta = \pm\pi/2$, the maximum value of the normal fracture circumferential stress along the axis $\theta = 0$ is given by the expression

$$\sigma_\theta(r, 0) = p[1 + (1/2)(a/r)^2 + (3/2)(a/r)^4]. \tag{1.11}$$

As in the previous case, the elementary fracture cell is a segment; therefore, according to the structural approach, the criterial relation is written as

$$\frac{1}{d}\int\limits_a^{a+d} \sigma_\theta(r, 0)\, dr = \sigma_c.$$

Calculation of the integral in the last equality yields

$$\frac{p_*}{\sigma_c} = \frac{1}{1 - (\Theta/2)(\Theta/(\Theta+1) - 1) - (\Theta/2)[(\Theta/(\Theta+1))^3 - 1]}, \tag{1.12}$$

where $\Theta = a/d$. As $\Theta \to 0$, we have $p_* = \sigma_c$, and as $\Theta \to \infty$, we have $p_*/\sigma_c = 1/3$.

In the problem of a rectilinear central crack, the expression for the structural parameter d is obtained by comparing the critical loads calculated by the structural criterion and the Irwin criterion. We analyze the possibility of using this expression in the present problem.

Results of experiments in which the critical load p_* was determined for uniaxial extension of plates with a circular hole of diameter 0.7–16.0 mm are given in [11]. A plate of dimensions $120 \times 400 \times 2$ mm (the dimensions were chosen such that the edge effects were negligible) was made of SCh 12-28 gray cast iron with mechanical properties $\sigma_c = 170$ MPa, $K_{\rm Ic} = 14$ MPa$\cdot$m$^{1/2}$, $\nu = 0.3$, and $E = 100 \cdot 10^3$ MPa. For this material, the structural fracture parameter d determined according to (1.4) is 4.3 mm.

Figure 2 shows a curve of the critical load on the hole radius a calculated by formula (1.12) (curve 1). It is evident that the results of calculation using the structural approach are in good agreement with experimental

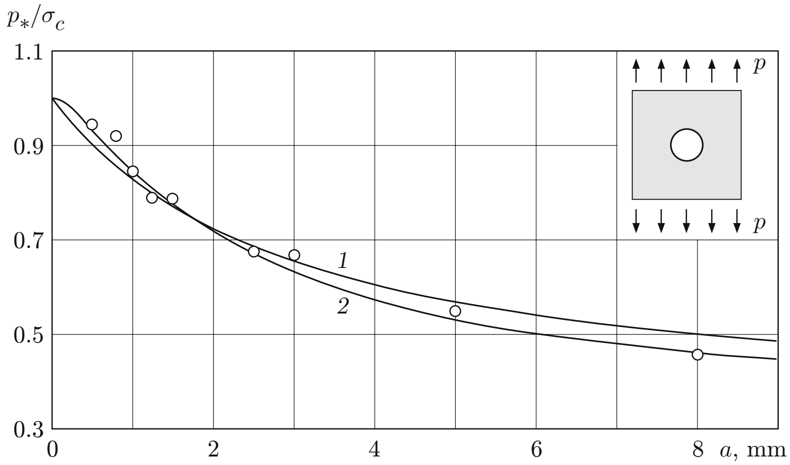

Fig. 2

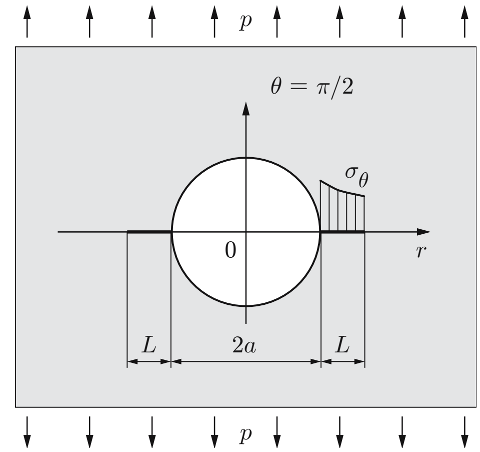

Fig. 3

Fig. 2. Critical load versus hole radius: 1) structural criterion; 2) Leonov–Rusinko criterion; the points refer to the experimental data (gray cast iron).

Fig. 3. Geometry of the fictitious crack in a plate with a hole.

data. Figure 2 gives a similar curve (curve 2) plotted using the criterion proposed by Leonov and Rusinko [12]: $p_* = \sigma_c/k$, where k is the macrostress concentration factor:

$$k = \frac{2\nu\alpha^2}{(1+\nu)(1+\alpha)^2(1+2\alpha+2\alpha^2)} + \frac{3+11\alpha+25\alpha^2+40\alpha^3+42\alpha^4+24\alpha^5+8\alpha^6}{(1+2\alpha+2\alpha^2)^3}. \tag{1.13}$$

Here $\alpha = \rho/a$ (ρ is the structural characteristic of the material that takes into account its microinhomogeneity). The theoretical value of the parameter ρ was obtained in [12]:

$$\rho = \beta E\gamma/\sigma_c^2. \tag{1.14}$$

Here the parameter β depends only on Poisson's ratio:

$$\beta = [4\nu\sqrt{1+\sqrt{2}} + (3-4\nu)\sqrt{2} - 1]^2/[4\pi(1+\sqrt{2}\,)(1-\nu^2)]. \tag{1.15}$$

In (1.15), the coefficient β varies in the range from 0.347 to 0.545 for $\nu = 0$–0.5.

For gray cast iron, the theoretical value of the structural parameter ρ calculated by formula (1.14) is 1.3 mm. However, the best agreement with the experimental data is obtained for $\rho = 1.4$ [11].

As $a \to 0$, the stress concentration factor k tends to the theoretical value $k = 3$. At the same time, as follows from (1.13), as $a \to \infty$, we have $k \to 1$, which corresponds to the strength of the defect-free material ($p_* \to \sigma_c$).

A different approach to solving this problem is applied in [6]. A so-called fictitious crack is introduced to estimate the fracture load p_* for the uniaxial extension of a plate with a circular hole of radius a. This is due to the need to have any characteristic with the dimension of length to compare it with the hole size. A fictitious crack of length L with origin on the boundary of the hole is located along the dangerous section (Fig. 3).

We assume that the edges of the fictitious crack are subjected to a stress $\sigma_\theta(r,0)$ distributed according to (1.11). If the stress is specified at the crack edges, the corresponding stress intensity factor can be found from the well-known formula

$$K_{\rm I} = \frac{p}{\sqrt{\pi L/2}} \int\limits_a^{a+L} \sigma_\theta(r,0)\sqrt{\frac{r-a}{L-(r-a)}}\,dr. \tag{1.16}$$

Substitution of σ_θ from (1.11) into (1.16) yields

$$K_{\rm I} = p\sqrt{\pi L/2}[1 + (1/2)(1+\vartheta)^{-3/2} + (3/2)(1+\vartheta)^{-7/2}(1+\vartheta/2+\vartheta^2/8)], \tag{1.17}$$

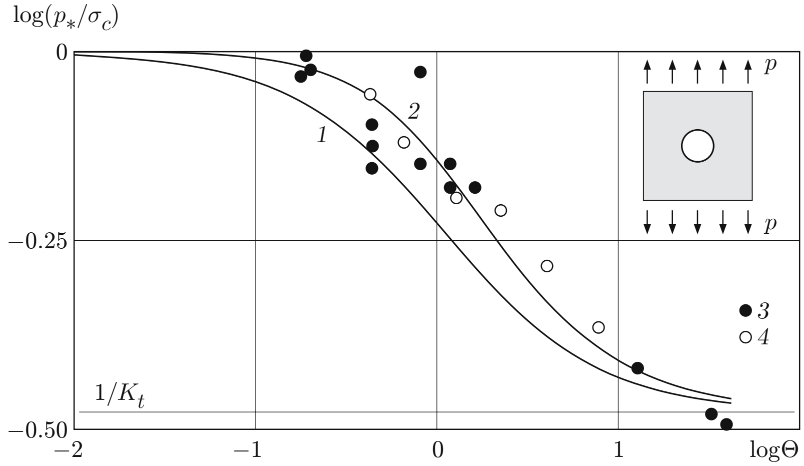

Fig. 4. Critical load versus hole radius: 1) structural criterion (1.12); 2) fictitious crack (1.19); 3) experimental data [6] for epoxy–carbon fiber; 4) the same for epoxy–glass fiber.

where $\vartheta = L/a$. As $\vartheta \to 0$, the expression in square brackets (1.17) becomes the stress concentration factor K_t. As $\vartheta \to \infty$, we obtain a plate without a hole with strength σ_c and a fictitious crack, which begins "to propagate" as $K_{\rm I} \to K_{{\rm I}c}$. Thus, in the limiting case, we set $\vartheta \to \infty$, $p \to \sigma_c$, and $K_{\rm I} \to K_{{\rm I}c}$. Then, relation (1.17) implies

$$L = 2K_{{\rm I}c}^2/(\pi\sigma_c^2). \qquad (1.18)$$

The right side of expression (1.18) is identical to the expression for the structural parameter d (1.4). Thus, in the structural approach, a fictitious crack of length L is equivalent to a prefracture zone of length d.

Assuming that the equality $\sigma_c = K_{{\rm I}c}\sqrt{2/(\pi L)}$ following from (1.18) is satisfied as $p \to p_*$, from (1.17) we obtain the critical load. For convenience of comparison with the experimental results and the value of the fracture load based on the structural approach (1.12), we introduce the dimensionless parameter $\Theta = a/L$. Then, from (1.17) we finally obtain

$$\frac{p_*}{\sigma_c} = \frac{1}{1 + (1/2)(1+1/\Theta)^{-3/2} + (3/2)(1+1/\Theta)^{-7/2}(1 + 1/(2\Theta) + 1/(8\Theta^2))}. \qquad (1.19)$$

A curve of the critical load versus relative radius of the hole in the plate is shown in logarithmic coordinates in Fig. 4. The experimental data on the fracture of composite plates with a circular hole are taken from a paper [6], which contains references to original papers.

From Fig. 4 it follows that over the entire range of hole sizes, the structural approach gives lower values of the maximum load than the fictitious crack method, which, however, contributes to the safety factor of the structure. The maximum difference is 21.4% at $\Theta = 0.85$.

Thus, in the present problem, as in the problem of extension of a plane with a central crack, the structural parameter d can also be determined, as a first approximation, from formula (1.4).

2. Two-Dimensional Fracture Criterion. We consider the axisymmetric problem of the extension of an elastic space with a disk-shaped crack of radius a subjected to a uniform load p at infinity. In cylindrical coordinates (r, θ, z), the asymptotic form of the fracture stress on the continuation of the crack is written as

$$\sigma_z = K_{\rm I}/\sqrt{2\pi(r-a)} + O(1), \qquad r - a \to 0, \quad r > a, \qquad (2.1)$$

where

$$K_{\rm I} = 2p\sqrt{a/\pi}\,. \qquad (2.2)$$

The accurate representation of the tensile stress σ_z on the continuation of the crack is also known:

$$\sigma_z = -(2p/\pi)[\arcsin(a/r) - a/\sqrt{r^2 - a^2}\,] + p, \qquad z = 0, \quad r > a. \qquad (2.3)$$

Applying the Irwin criterion to (2.2), we obtain

$$p_* = (1/2)K_{{\rm I}c}\sqrt{\pi/a}\,. \qquad (2.4)$$

Let us apply the structural criterion. Assuming that the elementary fracture cell has the shape of a ring sector [$a \le r \le a + d$ and $-d/(2a) \le \theta \le d/(2a)$] with area $S = d(2ad + d^2)/(2a)$, we write the fracture criterion as follows:

$$\frac{2}{2ad + d^2} \int_a^{a+d} \sigma_z(r) r\, dr = \sigma_c. \tag{2.5}$$

Applying (2.5) to the asymptotic solution (2.1) and performing integration in view of (2.2), we obtain the critical load. Equating its value to (2.4), we obtain the dependence of the fracture toughness $K_{\rm Ic}$ on the strength limit σ_c:

$$K_{\rm Ic} = \sigma_c \sqrt{\frac{\pi d}{2} \frac{1 + d/(2a)}{1 + d/(3a)}}. \tag{2.6}$$

The structural fracture parameter is now determined as the root of the cubic equation (2.6) and depends not only on the strength constants of the material but also on the crack radius. We denote this parameter by d_0.

As follows from (2.6), $d_0 \to d$ as $d_0/a \to 0$. Passing to the limit as $a/d_0 \to 0$, we obtain $d_0(0) = 4d/9$. Thus, the structural parameter d_0 can be treated as a generalization of the parameter d, and d as an asymptotic (as $d_0/a \to 0$) or degenerate case d_0.

The critical load determined from the exact solution (2.3) is equal to

$$\frac{p_*}{\sigma_c} = \frac{\pi(1 - \eta_0^2)}{2(\arccos \eta_0 + \eta_0 \sqrt{1 - \eta_0^2}\,)}.$$

Here $\eta_0 = a/(a + d_0)$, where $0 \le \eta_0 \le 1$, and $d_0 = d_0(a)$ is the root of Eq. (2.6) in which d needs to be replaced by d_0.

In this problem, one can also use the one-dimensional version of the fracture criterion

$$\frac{1}{d} \int_a^{a+d} \sigma_z(r)\, dr = \sigma_c. \tag{2.7}$$

We introduce the dimensionless parameter $\eta = a/(a + d)$ $(0 \le \eta \le 1)$. Then,

$$p_*/\sigma_c = \pi(1 - \eta)/(2 \arccos \eta).$$

In the case of the one-dimensional fracture criterion (2.7), the value of p_* is slightly smaller than that in the two-dimensional criterion (2.5).

The critical load can also be determined [9] by the Griffith criterion

$$p_*/\sigma_c = \sqrt{2a_*/a}$$

and by the critical crack opening criterion

$$\frac{p_*}{\sigma_c} = \begin{cases} 1, & a < a_*, \\ \sqrt{2a_*/a}\sqrt{1 - a_*/(2a)}, & a \ge a_*, \end{cases} \tag{2.8}$$

where

$$a_* = \pi E \delta_c / (8(1 - \nu^2)\sigma_c);$$

$\delta_c = 2\gamma/\sigma_c$ is the critical crack opening.

Figure 5 gives curves plotted with allowance for the equivalence of the power and energetic fracture criteria [equality (1.9)]. An analysis shows that for integration over a ring (the two-dimensional version) and for integration over a segment (one-dimensional version), the difference between the values of p_*/σ_c determined using the structural criterion does not exceed 1.7%. However, there is a significant difference between the critical loads determined by the critical crack opening criterion and by the structural criterion for cracks of small size (the maximum difference is 18.5%).

Condition (2.8) implies that for a disk-shaped crack, the size a_* is the limiting one, i.e., cracks of radius $a < a_*$ can be ignored and the material can be considered free from defects. This conclusion, however, does not

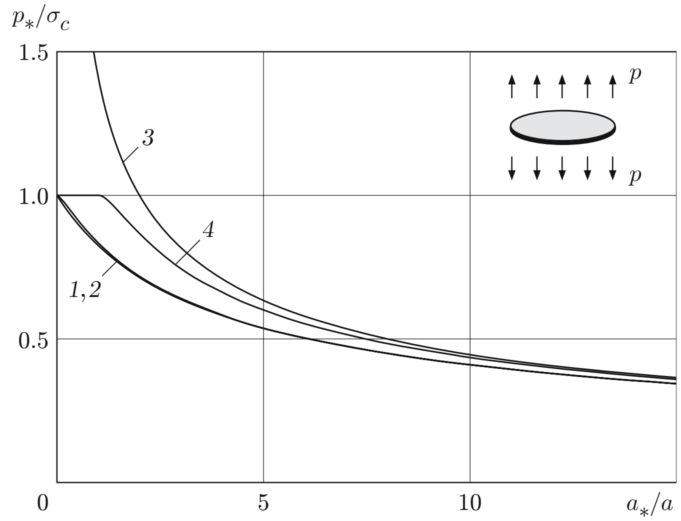

Fig. 5. Critical load versus radius of a disk-shaped crack: 1) one-dimensional structural criterion; 2) two-dimensional structural criterion; 3) Griffith (Irwin) criterion; 4) critical crack opening criterion.

follow directly from the solution of the problem: the exact solution is only the second line in (2.8), and in the range $a_*/2 \le a \le a_*$, the critical loading decreases from σ_c to zero, and $dp_*/da = 0$ for $a = a_*$. In [9], solution (2.8) was constructed using additional assumptions, which limited the solution to the case $a \ge a_*$.

From a physical point of view, the existence of such small cracks that "do not influence" the strength of disk-shaped cracks is explained in [9] by the fact that the propagation of a crack of diameter $2a < 2a_*$ is energetically unfavorable since in the case of crack opening, the amount of elastic energy released is smaller than the amount of the effective surface energy accumulated on the free surfaces of the crack. It is not clear, however, why such limiting sizes are not found, for example, in the case of a central crack (planar problem).

As an example of a smooth stress concentrator, we consider the problem of an elastic space containing a spherical cavity of radius p and subjected to uniaxial uniform extension by a stress a. The extension is performed in the z direction in cylindrical coordinates (r, θ, z). The fracture stress σ_z in the plane $z = 0$ is expressed as

$$\sigma_z(r) = p\Big(1 + \frac{4-5\nu}{2(7-5\nu)}\,\frac{a^3}{r^3} + \frac{9}{2(7-5\nu)}\,\frac{a^5}{r^5}\Big). \tag{2.9}$$

In this problem, the two-dimensional structural fracture criterion has the same form as in the case of a disk-shaped crack (2.5). Substitution of expression (2.9) into the criterial relation (2.5) yields

$$\frac{p_*}{\sigma_c} = \frac{1+\eta}{1+\eta(1+\eta)(1+3\eta^2/(7-5\nu))}, \tag{2.10}$$

where $\eta = a/(a+d)$.

Using the one-dimensional structural criterion (2.7), we obtain

$$\frac{p_*}{\sigma_c} = \frac{1}{1+\eta(1+\eta)[1+3(1+\eta^2)/(2(7-5\nu))]/4}. \tag{2.11}$$

From (2.10) and (2.11), it follows that $p_* \to \sigma_c$ as $\eta \to 0$ (an infinitesimal defect) and

$$\frac{p_*}{\sigma_c} = \frac{2(7-5\nu)}{3(9-5\nu)} = \frac{1}{K_t}$$

as $\eta \to 1$ (a infinitely large defect); K_t is the stress concentration factor. Thus, solutions (2.10) and (2.11) are asymptotically equivalent, i.e., coincide as $\eta \to 0$ and $\eta \to 1$. However, for other practically important values of the parameter η, these solutions differ (Fig. 6).

Let us estimate the limit load using the fictitious crack method. For this, we consider an arbitrary section through the center of the cavity along the Oz axis. For such a fictitious crack of length L, which begins on the

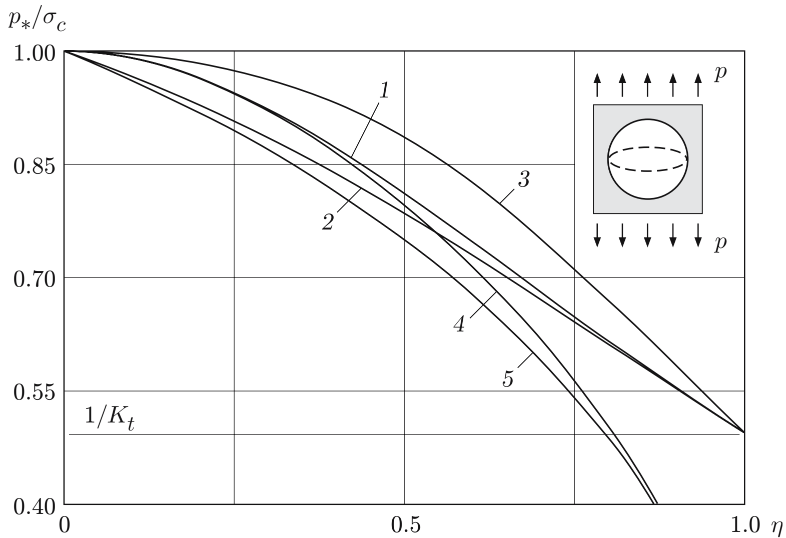

Fig. 6. Critical load versus the parameter η for a spherical cavity ($\nu = 0.25$): 1) two-dimensional structural criterion (2.5); 2) one-dimensional structural criterion (2.7); 3) fictitious crack method (2.12); 4) disk-shaped crack [two-dimensional structural criterion (2.5)]; 5) disk-shaped crack [one-dimensional structural criterion (2.7)].

boundary of the cavity and is located in the dangerous section $z = 0$, the stress intensity factor is determined from expression (1.16) in which instead of $\sigma_\theta(r, 0)$ we use $\sigma_z(r, 0)$. Proceeding further as in the problem of a circular hole, we obtain

$$p_*/\sigma_c = 1/f(\eta), \qquad \eta = a/(a + d), \quad 0 \le \eta \le 1, \tag{2.12}$$

where

$$f(\eta) = 1 + \frac{4 - 5\nu}{8(7 - 5\nu)}\Big(3 + \frac{1}{\eta}\Big)\Big(\frac{1}{\eta}\Big)^{-5/2}$$

$$+ \frac{9}{128(7 - 5\nu)}\Big[64 + 5\Big(\frac{1}{\eta} - 1\Big)^3 + 24\Big(\frac{1}{\eta} - 1\Big)^2 + 48\Big(\frac{1}{\eta} - 1\Big)\Big]\Big(\frac{1}{\eta}\Big)^{-9/2}.$$

Dependence of (2.12) is presented in Fig. 6. It is evident that the fictitious crack method gives a result closer to the result obtained using the two-dimensional structural criterion.

In [13], it was assumed that the effect of a spherical pore on the strength of an alloy based on tungsten carbide and cobalt (WC–10% Co) is equivalent to the effect of a disk-shaped crack of the same diameter. This hypothesis is confirmed by the estimates obtained above using the structural approach (curves 4 and 5 in Fig. 6). From Fig. 6, it follows that for spherical pores of small sizes (compared to d), there is good agreement between the critical loads for a circular crack and a spherical cavity. In this case for the two-dimensional structural criterion, the values of the critical loads almost coincide. For both defects in the vicinity of the point $\eta = 0$, we have the estimate

$$p_*/\sigma_c = 1 - \eta^2 + O(\eta^3).$$

For $a \le 1.6d$, the relative difference between the critical loads does not exceed 5% for a material with $\nu = 0.25$. For smaller values of Poisson's ratio, this difference is slightly larger and at for $\nu = 0.5$, it is the smallest. The use of the two-dimensional structural criterion in the problem considered may be preferred.

To obtain a quantitative estimate of the dimensional critical load, it is necessary to specify the value of the structural parameter d. In the problem in question, we use the solution for a disk-shaped crack as an approximate version. As noted above, the maximum value of the structural parameter is equal to d and is determined from formula (1.4) and its minimum value is equal to $4d/9$. Indirect comparison with experimental data [13] indicates that the best agreement between the theoretical and experimental data is obtained if the structural parameter d

is calculated in the same way as in the planar problem, i.e., by formula (1.4). We note that the minimally safe diameter of a spherical pore was determined in [14] using the gradient approach and the structural parameter d calculated by formula (1.4); for WC–10% Co cermet, it equals $2.237d$.

Conclusions. The analysis of experimental data and theoretical estimates of the strength of materials with sharp notches and smooth stress concentrators suggests that in planar problems, the structural fracture parameter d can be determined using two standard characteristics: the tensile strength limit σ_c and the fracture toughness $K_{\rm Ic}$. For spatial defects in an uniaxial tension field with a circular interface between the boundary conditions, it is reasonable to determine the critical load using the two-dimensional version of the structural fracture criterion.

The analysis showed that the structural criterion provides a qualitative description of the dependence of the strength of a solid on the stress concentrator sizes in both planar and spatial problems. To obtain a more reliable quantitative estimate of critical loads, it is required to specify the value of the structural fracture parameter d based on experimental data.

REFERENCES

1. S. V. Suknev, "Criterion of local strength," *Probl. Prochn.*, No. 4, 108–124 (2004).
2. H. Neuber, *Kerbspannunglehre: Grunglagen für Genaue Spannungsrechnung*, Springer-Verlag, Berlin (1937).
3. V. V. Novozhilov, "Necessary and sufficient criterion of brittle strength," *Prikl. Mat. Mekh.*, **33**, No. 2, 212–222 (1969).
4. V. I. Smirnov, "Two-criterion model for the fracture of a brittle space with a disk-shaped crack," in: *Deformation and Fracture Mechanisms of Promising Materials*, Proc. 35 Workshop "Urgent Problems of Strength" (Pskov, September 15–18, 1999), Pskov (1999), Part 1, pp. 66–68.
5. Yu. V. Petrov, "'Quantum' macromechanics of dynamic fracture of solids," Preprint No. 139, Inst. of Problems of Mechanics, Russian Academy of Sciences, St. Petersburg (1996).
6. J. Tirosh, "On the tensile and compressive strength of solids weakened (strengthened) by an inhomogeneity," *Trans. ASME, J. Appl. Mech.*, **44**, No. 3, 449–454 (1977).
7. S. E. Kovchik, "Some experimental studies of crack propagation in glass plates," in: *Problems of Mechanics of Real Solids* (collected scientific papers) [in Russian], No. 2, Naukova Dumka, Kiev (1964), pp. 172–176.
8. N. N. Davidenkov and A. N. Stavrogin, "On the strength criterion for brittle fracture and a planar stress state," *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk*, **9**, 101–109 (1954).
9. V. V. Panasyuk, *Limit Equilibrium of Brittle Solids with Cracks* [in Russian], Naukova Dumka, Kiev (1968).
10. M. Ya. Leonov, *Mechanics of Deformation and Fracture* [in Russian], Ilim, Frunze (1981).
11. S. Ya. Yarema and L. V. Ratych, "Brittle fracture of samples with stress concentrators," in: *Stress Concentration* (collected scientific papers) [in Russian], No. 1, Naukova Dumka, Kiev (1965), pp. 338–343.
12. M. Ya. Leonov and K. N. Rusinko, "Macrostresses of an elastic solid," *J. Appl. Mech. Tech. Phys.*, No. 1, 104–110 (1963).
13. A. Nordgren and A. Melander, "Influence of porosity on strength of WC–10% Co cemented carbide," *Powder Metallurg.*, **31**, No. 3, 189–200 (1988).
14. M. A. Legan, "Correlation of local strength gradient criteria in a stress concentration zone with linear fracture mechanics," *J. Appl. Mech. Tech. Phys.*, **34**, No. 4. 585–592 (1993).